DISTRIBUTED LAG MODELS AND THE EFFECTS OF SOCIOECONOMIC CHANGE ON FERTILITY

William Ray Arney
University of Colorado

Interest in the relationship between socioeconomic and demographic change has developed because of the widespread attention given the theory of demographic transition (Thompson, 1929, 1944; Notestein, 1945). Briefly, the theory states that there are three stages in the history of a population: (1) the population is stable and a regime of high birth and death rates prevails, (2) the population undergoes a transition in which death rates decline followed by a decline in fertility, and (3) the population reaches a fairly stable state which is characterized by low birth and death rates. The relationship between socioeconomic and demographic change becomes most important at the second or transitional stage of development. It has been suggested that technology to control mortality is applied more ubiquitously and rapidly under conditions of high urbanization and industrialization. This, in connection with the high fertility rates, causes rapid population growth. Fertility responds with a downward trend, according to speculation, because high fertility handicaps the population "in their effort to take advantage of the opportunities being provided by the emerging economy" (Davis, 1963: 352).

Students of population dynamics are quick to point out that the theory of demographic transition initially was simply an empirical generalization with no real rationale (Gutman, 1960). However, in more recent work we find several schemes which attempt to provide rationales for the observations. For example, Cowgill begins with the Malthusian supposition that a population will grow at a geometric rate until it approaches a size which begins to affect resource and space availability. He believes that most populations "at any given time" have reached this point and have achieved "a condition of equilibrium characterized by a relatively stationary population" (1963: 271). His refinement of transition theory is contained in the assertion that technological advances increase the "carrying capacity of the environment" (presumably a function of resource and space availability) which leads to a period of population increase. He explicitly states:

> "Under conditions of industrialization, given the technology of birth and death control, there is a marked tendency for the technology of death control to be applied earlier and more extensively resulting in rapid population growth ..." (p. 272).

Generally congruent with Cowgill's propositions is the theory of economic and demographic interdependence developed by Frederiksen (1969). His model posits complex interaction among variables such as technological and socioeconomic development, production, "levels of nutrition, sanitation, health services, etc.," mortality, and fertility. With considerable simplification his theory can be stated as follows. With increasing technological development comes increasing health care distribution and implementation which causes a decrease in mortality. Decreased mortality helps decrease fertility by increasing survival probabilities of offspring and thereby decreasing the need for large families. This model views a decline in mortality, therefore, not as a cause of the population problem but as a necessary factor for the solution of it.

On the basis of these two important theoretical contributions we can identify one major factor which should be included in an explanation of changes in fertility. Urbanization brings about changes which are more conducive to mortality decline which can lead to a reduction of fertility. Therefore, at the most general level, one might expect urbanization and fertility to be inversely related.

A second set of factors which should theoretically influence fertility is derived from economic theories of fertility behavior. Generally, these variables influence fertility through mechanisms regulated by opportunity costs (Gronau, 1973; Mincer, 1963). It is thought that as the social status of women increases (in terms of education, economic earning power, and so forth) the costs of having children increases. That is, having children removes the opportunity to earn more money, or in some way other than raising children, to make use of the woman's time. Thus, variables related to the social and economic position of women in a society become an important factor in the analysis of fertility.

Unfortunately, data related to the employment of women of the form required for this study (long time series data) were unavailable. Therefore, it was decided to use a proxy which was though to be indicative of overall opportunity costs -- unemployment. It was hypothesized that as unemployment increased, opportunity costs associated with having children would decrease. As opportunity costs decrease, fertility should increase; so a direct relationship between unemployment and fertility was expected.

A third factor assumed to be affecting fertility was religious composition of a population. Heer and Boynton (1970) found that the second highest correlation in their study of data for counties in the United States was between fertility and the proportion of Roman Catholics in the diocese or archdiocese in which the county was located. Although the reproductive ideals and behavior of Roman Catholics seems to be changing it was thought that, particularly for the time period which the data for this study covers -- 1919 to 1967 -- an important variable to include in the analysis was the percentage of the population who were Roman Catholic. A direct relationship between the percentage of Roman Catholics in the population and fertility was expected.

PROCEDURE

Considerable effort has been devoted to the study of the effects of socioeconomic change on fertility. Previously, most studies have used cross-sectional data and various forms of regression analysis. Although this has the merits of data availability (at least more so than the time series data that will be used here) and extensive technical development, the cross-sectional approach is inadequate for two reasons. First, in order to make inferences concerning developmental processes it is necessary to infer a developmental dimension into the cross-sectional data or as Goldscheider (1971: 85) puts it, one must exercise an "evolutionary bias". He notes,

> "The foundation of prediction and control ... assumes that currently nonmodern nations will follow similar patterns of development experienced by currently modern nations, and that specific relationships between social, economic, political, and cultural variables on one hand and population variables on the other will be the same in the modernization process of developing societies as they were in the historical experience of developed societies."

The present study avoids this problem by examining the development of a single country as that development is reflected in historical time series. That is, the time dimension is considered explicitly and no time dependent variable need be inferred into the data or the results of the analysis.

Second, regression analysis requires a rather strict parametric structure. Heise (1969) calls this the problem of specification. The procedures used here to estimate models of fertility circumvents the specification problem at least to a degree. Also, depending on data collection procedures, the lag structure of a regression model can be restricted. For example, if data on divorces and marriages were collected from a number of "most recent censuses" the regression model which would result would relate the variables at only one point in time. This is clearly unrealistic since one would logically expect divorce rates to rise a certain number of years after an increase in marriages. In other words, divorces should lag marriages and a model of the relationship between these variables should reflect this. Even more realistically, since "length of marriage to divorce" is distributed over a number of years one should expect an increase in marriages in year x to cause an increase in divorces in years x, x+1, x+2, ... A model to describe this relationship is called a distributed lag model. In fact, Carlsson (1970) has demonstrated the utility of the distributed lag concept with his model of the relationship between fertility and marriages in nineteenth century Sweden.

In this study, time series data for the United States is used and spectral analysis is employed to estimate a distributed lag relationship between fertility on the one hand, and urbanization, unemployment, and Roman Catholic population on the other. Yearly data on each of the variables was collected from published sources. Urbanization was measured by the complement of the farm population which has been estimated annually by the U.S. Department of Agriculture. Estimates of the proportion of the population belonging to the Roman Catholic church were obtained from series H-538 of the Historical Statistics of the United States (Washington, D.C.: Government Printing Office, 1960; continuation, 1965) and various issues of the Statistical Abstract of the United States. Unemployment was taken from series D-47 of the Historical Statistics and issues of the Statistical Abstracts.

Births per 1,000 women aged 15-44 years (computed by summing birth rates by age of mother in five year age groups multiplied by five) was chosen as the dependent variable. Estimates of this variable are available on a yearly basis since 1909 from the Bureau of Vital Statistics. This indicator of fertility has the advantage over other measures (e.g., the crude birth rate) that an adjustment has been made for age composition. This is beneficial since there is an interaction between fertility and a population's age distribution.

The use of spectral analysis to estimate distributed lag models is reviewed in detail in other sources (Jenkins and Watts, 1968; Fishman, 1969; Hannan, 1963, 1965, 1967; Mayer and Arney, 1974). Due to space limitations it will be reviewed here with considerable brevity.

If we assume that a discrete time series $\{x_t\}$ is related to the series $\{y_t\}$ then a distributed lag relating the two has the form

$$y_t = \sum_{i=0}^{\infty} h_i\, x_{t-i} \, . \tag{1}$$

In order to estimate the collection of constants $\{h_i\}$ we must first make an assumption about and place certain restrictions on the series $\{x_t\}$ and $\{y_t\}$. First, it must be assumed that the series are realizations of discrete stochastic processes $\{X_t\}$ and $\{Y_t\}$. Second, in order to use spectral analysis the underlying stochastic processes must be covariance stationary. That is, the covariance structure of the processes must not be dependent on the ordering variable, t, which is usually taken to be time.

Under the above restrictions equation 1 can be considered to be a representation of the system which linearly relates the discrete covariance stationary processes $\{X_t\}$ and $\{Y_t\}$ as

$$Y_t = \sum_{i=0} h_i\, X_{t-i} + Z_t \, . \tag{2}$$

The Z_t in equation 2 are terms of a discrete white noise process. That is, $\{Z_t\}$ is a discrete stochastic process in which all terms are independent of one another and each of the terms has the same distribution. The set of constants $\{h_i\}$ is called the impulse response function of the system. The problem becomes one of solving equation 2 for the impulse response function $\{h_i\}$.

A theorem due to the mathematician Norbert Weiner provides an initial step toward the solution of this problem. The Weiner-Hopf theorem states that the impulse response function in equation 2 which minimizes the mean square error of the linear prediction must also satisfy the relation

$$\gamma_{xy}(u) = \sum_{i=0} h_i\, \gamma_{xx}(u-i) \tag{3}$$

where the γ_{xx} and γ_{xy} are the autocovariance function of $\{X_t\}$ and the cross-covariance function of $\{X_t\}$ and $\{Y_t\}$, respectively. Using equation 3 and spectral functions it will be possible to solve for $\{h_i\}$.

The spectrum of the process $\{X_t\}$, denoted $\Gamma_{xx}(f)$, is the Fourier transform of the autocovariance function of $\{X_t\}$. Similarly, the cross-spectrum of the processes $\{X_t\}$ and $\{Y_t\}$, denoted $\Gamma_{xy}(f)$, is the Fourier transform of the cross-covariance function of $\{X_t\}$ and $\{Y_t\}$. If we let H(f) be the Fourier transform of the impulse response function then, due to the mathematical nature of Fourier transforms the frequency domain representation of equation 3 is

$$(4) \qquad \Gamma_{xy}(f) = H(f)\, \Gamma_{xx}(f).$$

H(f) is called the frequency response function of the system. Equation 4 can now be solved for H(f) by

$$(5) \qquad H(f) = \frac{\Gamma_{xy}(f)}{\Gamma_{xx}(f)} .$$

Since H(f) is the Fourier transform of the impulse response function, $\{h_i\}$, the impulse response function can be found by taking the inverse transform of H(f).

A system with multiple inputs $\{X_{k,t}, k = 1, 2, \ldots n\}$ and a single output $\{X_{n+1,t}\}$ can be represented by

$$(6) \qquad X_{n+1,t} = \sum_{k=1}^{n} \sum_{i=0}^{\infty} h_{k,i}\, X_{k,t-i} + Z_t$$

Following reasoning similar to that for the bivariate can it can be shown that if $G_{n+1}(f)$ is a vector of cross-spectra between each input and the output series, $G_{nn}(f)$ is a square matrix of spectra and cross-spectra among all the inputs, and $H_{n+1}(f)$ is a vector of partial frequency response functions, then

$$(7) \qquad H_{n+1}(f) = G_{n+1}(f)\, G_{nn}^{-1}(f)$$

provided the inverse of $G_{nn}(f)$ exists. The partial impulse response functions $\{h_{k,t}, k = 1, 2, \ldots, n\}$ can be found by taking the inverse Fourier transforms of the series of vectors $H_{n+1}(f)$.

There are two major adjustments of the data which must be made before spectral analysis can be used to estimate the coefficients in a model like equation 6. The data must be filtered to approximate covariance stationarity and then the independent series must be aligned with the dependent series to avoid biasing the spectral functions.

Generally, a first or second difference filter is sufficient to remove severe non-stationary components of a time series. A first order difference filter has the form

$$(8) \qquad x_t^* = x_t - x_{t-1}$$

where $\{x_t^*\}$ is the filtered series, and a second difference filter is merely the result of applying a first difference filter twice

$$(9) \qquad x_t^* = (x_t - x_{t-1}) - (x_{t-1} - x_{t-2}) = x_t - 2x_{t-1} + x_{t-2} .$$

These two filters can be thought of as the discrete analogs of derivatives. A second difference filter was necessary to achieve approximate covariance stationarity in the fertility and Roman Catholic population series. The non-stationarity evident in the urbanization and unemployment series was reduced by a first difference filter. The criterion used to judge approximate stationarity was a noticable reduction in the low frequency variation of the series as shown by the spectrum of the filtered series.

After filtering the data one must align the filtered independent series with the filtered dependent series. This is a requirement of cross-spectral analysis, but it also allows one to at least make an "educated guess" about the structure of the system with which one is working. That is, alignment requires the specification of a lead or lag relationship between two series.

It will be recalled that in order to estimate the cross-spectrum of two processes the cross-covariance function must be estimated first. The estimate of the cross-covariance function used in this paper is

$$(10) \qquad \gamma_{xy}(u) = \frac{1}{n-u} \sum_{i=1}^{n-u} (x_i - \bar{x})\, (y_{i+u} - \bar{y})$$

where $\bar{x}$ and $\bar{y}$ are the sample means of the series $\{x_t\}$ and $\{y_t\}$ respectively. The cross-covariance function will peak at a value of u, u', where the covariance between one series and the other series shifted u' units is greatest. If the series are not aligned, i.e., adjusted so that the greatest covariance occurs at zero lags, the estimated spectral functions suffer considerable bias. For example, inspecting the cross-covariance function between the filtered urbanization and fertility series it was found that urbanization lagged fertility by four years. Accordingly the filtered urbanization series was transformed by

$$(11) \qquad u_t^{**} = u_{t-4}^*$$

and all spectral and cross-spectral functions were estimated for the series $\{u_t^{**}\}$ and $\{f_t^*\}$. Similarly the following adjustments of the other independent series were required

$$(12) \qquad \text{for unemployment: } v_t^{**} = v_{t-1}^*$$

$$(13) \qquad \text{for Roman Catholic pop.: } r_t^{**} = r_{t-3}^*$$

where a single asterik denotes the filtered series and a double asterik denotes the filtered, aligned series.

RESULTS

Using the procedure outlined above a distributed lag model between the filtered and aligned series was estimated. The partial impulse response functions were obtained by applying the inverse Fourier transformation to the partial frequency response functions. The partial impulse response functions appear in Table I.

Model I

One problem in constructing a multivariate distributed lag model is how to choose the number of terms from each partial impulse response function to include in the model. One solution to this problem involves the inspection of the cross-covariance function between two series. If the cross-covariance function drops off rapidly after p lags, it is probable that p terms of that impulse response function should be included in the model.

Using this technique it was decided that one term of each of the first two partial impulse response functions (urbanization and unemployment) and five terms of the third partial impulse response function should be included in the model.

This resulted in the equation

$$(14)\quad f_t^* = -4.75\, u_t^{**} + .78\, v_t^{**} - .039\, r_t^{**} - 3.92\, r_{t-1}^{**} + 3.95\, r_{t-2}^{**} - .17\, r_{t-3}^{**} - 1.81\, r_{t-4}^{**} \quad .$$

Equation 14 will be called Model I. It explains only 11% of the filtered fertility series suggesting that this model is rather inadequate.

Model II

An alternative method for selecting terms of the partial impulse response functions to enter the model is a computer search to meet some criterion. In this case, variance explained by a linear projection was maximized. The model constructed in this way is

$$(15)\quad f_t^* = -4.75\, u_t^{**} + .78\, v_t^{**} - 1.23\, v_{t-4}^{**} + .13\, v_{t-5}^{**} + .45\, v_{t-6}^{**} - .34\, v_{t-7}^{**} - .039\, r_t^{**} - 3.92\, r_{t-1}^{**} + 3.95\, r_{t-2}^{**} - .17\, r_{t-3}^{**} - 1.81\, r_{t-4}^{**} \,.$$

This model is certainly a better predictor of fertility since it explains approximately 39% of the variance in the filtered fertility series. One method of evaluating the adequacy of this model is to compare the original spectrum of the filtered fertility series to the spectrum of the residual series. If the residual series were purely random the residual spectrum would be essentially flat (Jenkins and Watts, 1968: 224-225). Figure I shows that considerable flattening has occurred even though the residual spectrum is not completely flat.

Notice that equation 15, Model II, is similar to Model I in that it has one term of the partial impulse response function associated with the urbanization series and five term of the "Roman Catholic population" partial impulse response function. However, Model II differs significantly from Model I with respect to the unemployment coefficients. Terms corresponding to lags of zero, four, five, six, and seven years enter the model. This is especially curious since the terms of the partial impulse response function corresponding to lags of one, two, and three years are certainly not zero.

Is it possible to reconcile this finding? It is possible that the secondary delay is due to some form of recursion, i.e., unemployment exerting an influence on fertility indirectly through another variable. To check this speculation the cross-covariance functions between the unaligned unemployment series and the other two independent series were examined. It was found that a very strong relationship existed between urbanization and unemployment lagged one year. Recall that urbanization required an adjustment of four years to achieve proper alignment with fertility. These four years together with the lag of one year of unemployment behind urbanization suggests an indirect effect of unemployment on fertility occurring with a lag of five years through the intervening variable urbanization. This indirect effect occurs at approximately the same lag as suggested by the structure of equation 15. In other words, a system of the form in Figure II seems to be operative here.

The total effect of unemployment on fertility can be found by adding the convolution of the distributed lag relating unemployment to urbanization and the distributed lag relating urbanization to fertility with the direct effect of unemployment on fertility. By methods described above it was found that the best linear predictor of urbanization based on unemployment was

$$(16)\quad u_t^* = -.1615\, v_{t-1}^*$$

This, when convoluted with the term of the partial impulse response function associated with the urbanization series which was included in Model II yields an equation which is quite similar to Model II.

$$(17)\quad f_t^* = -\,4.75\, u_t^{**} + .78\, v_t^{**} - .768\, v_{t-4}^{**} - .039\, r_t^{**} - 3.92\, r_{t-1}^{**} + 3.95\, r_{t-2}^{**} - .17\, r_{t-3}^{**} - 1.81\, r_{t-4}^{**} \,.$$

As can also be seen the magnitude of the coefficient for the v_{t-4}^{**} term is similar to the same coefficient in Model II. This secondary analysis lends support to the above speculation concerning the nature of the operative system.

DISCUSSION

If we remove the alignment and formulate equation 15 in terms of differencing operators Δ we obtain

$$(18)\quad \Delta f_t = \Delta f_{t-1} - 4.75\, \Delta u_{t-4} + .78\, \Delta v_{t-1} - 1.23\, \Delta v_{t-5} + .131\, \Delta v_{t-6} + .45\, \Delta v_{t-7} - .039\, \Delta r_{t-3} - 3.89\, \Delta r_{t-4} + 7.88\, \Delta r_{t-5} - 4.12\, \Delta r_{t-6} - 1.64\, \Delta r_{t-7} + 1.81\, \Delta r_{t-8}$$

As can be seen the most immediate effects of unemployment and urbanization are in the expected directions. Urbanization has a strong negative effect on fertility change after a lag of four years. The initial effect of unemployment on changes in fertility is positive and occurs at a lag of one year lending support to the opportunity cost argument concerning the influence of economic change on fertility. The longer term effects of unemployment on fertility are decidedly negative which suggests that in the long run income effects come into play. Perhaps the most confusing distributed lag relationship is that between Roman Catholic population and fertility. The strongest influence is positive and occurs at a lag of five years. However, the positive effect is offset somewhat by the negative effects at lags of three, four, six, and seven years.

In future papers the analysis performed here will be extended in several ways. First, other than substantive uses there is a question concerning possible uses of the types of models developed here. It is possible that such models could be used for purposes of projection and prediction of future trends in fertility on the basis of socioeconomic change. This will be explored. Second, the present model is being extended to include constructed recursive effects. In other words, instead of detecting recursion as was done in the present work recursive effects of variables will be "built in" by the investigator in theoretically meaningful ways. For example, it is thought that urbanization has an indirect effect on fertility through its influence on health care distribution and that variable's subsequent effect on infant mortality. This, then, leads to a reduction in fertility.

Much work remains to be done in this area. The present study is preliminary at best. We proceeded by effectively ignoring problems of data consistency over time and other methodological difficulties. Essentially, we were trying to answer the question of whether this approach can be useful at all in sociology. Inasmuch as it provides a new framework (with its own theoretical implications) for the analysis of data of a form not typically used by sociologists, the answer to the question is that the method appears useful, but further research may serve to qualify this initial response.

REFERENCES

Carlsson, Gosta.
1970 "Nineteenth-century fertility oscillations." Population Studies 24: 413-422.

Cowgill, Donald O.
1963 "Transition theory as general population theory." Social Forces 41: 270-274.

Davis, Kingsley.
1963 "The theory of change and response in modern demographic history." Population Index 29: 345-365.

Fishman, George S.
1969 Spectral Methods in Econometrics. Cambridge, Mass.: Harvard University Press.

Frederiksen, Harald.
1969 "Feedbacks in economic and demographic transition." Science 166: 837-847.

Goldscheider, Calvin.
1971 Population, Moderization, and Social Structure. Boston: Little, Brown, and Company.

Gronau, Reuben.
1973 "The effect of children on the housewife's value of time." Journal of Political Economy 81: S168-S199.

Gutman, Robert.
1960 "In defense of population theory." American Sociological Review 25: 325-333.

Hannan, E. J.
1963 "Regression for time series." Pp. 17-37 in M. Rosenblatt (ed.), Proceedings of the Syposium on Time Series Analysis. New York: John Wiley.
1965 "The estimation of relationships involving distributed lags." Econometrica 33: 206-224.
1967 "The estimation of a lagged regression relation." Biometrika 54: 409-418.

Heer, David M., and John W. Boynton.
1970 "A multivariate regression analysis of differences in fertility of United States counties." Social Biology 17: 180-194.

Heise, David R.
1969 "Problems in path analysis and causal inference." Pp. 38-73 in E. F. Borgatta (ed.), Sociological Methodology 1969. San Francisco: Jossey-Bass.

Jenkins, Gwilym M., and Donald G. Watts.
1968 Spectral Analysis and Its Applications. San Francisco: Holden-Day.

Mayer, Thomas F., and William Ray Arney.
1974 "Spectral analysis and the study of social change." Pp. 309-355 in H. L. Costner (ed.), Sociological Methodology 1973. San Francisco: Jossey-Bass.

Mincer, Jacob.
1963 "Market prices, opportunity costs, and income effects." Pp. 67-82 in C. Christ, et. al (eds.), Measurement in Economics. Stanford: Stanford University Press.

Notestein, Frank W.
1945 "Population - the long view." Pp. 36-57 in T. W. Schultz (ed.), Food for the World. Chicago: University of Chicago Press.

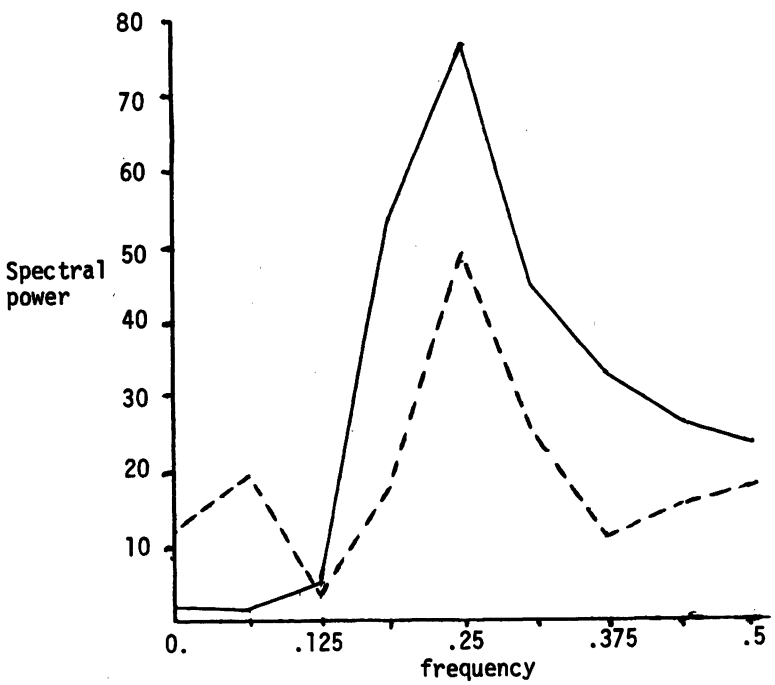
Thompson, Warren S.
1929 "Population." American Journal of Sociology 34: 959-975.
1944 Plenty of People. Lancaster, Pa.: Jacques Cattell Press.

TABLE I

Partial impulse response functions for a model of fertility based on the filtered, aligned urbanization, unemployment, and Roman Catholic population series.

lags	partial impulse response functions		
	urbanization	unemployment	Roman Catholic population
0	-4.754	.782	-.039
1	-3.845	-1.630	-3.929
2	2.226	.248	3.956
3	2.620	1.159	-.172
4	-1.154	-1.236	-1.815
5	-5.039	.131	1.954
6	3.321	.453	-1.183
7	.858	-.340	-1.370

Figure I: Original spectrum of the filtered fertility series and the residual spectrum derived from Model II.



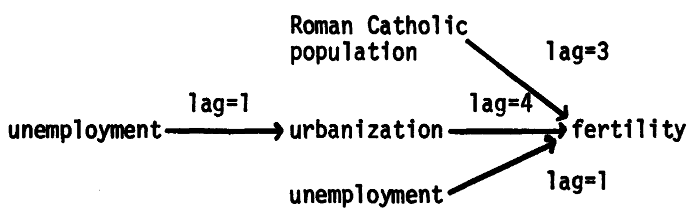


Figure II: Relationships among filtered urbanization, unemployment, Roman Catholic population, and fertility series.